



امتحان السداسي الأول 2024/2023

تمرين 1. (3pts)

في فضاء 3D ، احسب مايلي:

$$\delta_{im}\delta_{mn}\delta_{nj} , \delta_{ij}\delta_{jk}\delta_{ki} , \delta_{ii}\delta_{jj} , x_i x_i = r^2$$

تمرين 2. (3pts)

برهن أن: $\epsilon_{ijm}\epsilon_{klm} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}$

تمرين 3. (10pts)

لدينا $\vec{c} = c_k \vec{e}_k$ ، $\vec{b} = b_j \vec{e}_j$ ، $\vec{a} = a_i \vec{e}_i$
برهن أن:

$$a.(b \times c) = c.(a \times b) = b.(c \times a) -$$

$$a \times (b \times c) = (a \times c).b - (a \times b).c -$$

تمرين 4. (4pts)

برهن أن محدد المصفوفة [A]:

$$\det A = \epsilon_{ijk} A_{i1} A_{j2} A_{k3} -$$



تمرين 1 (3pts)

حساب العناصر

0,75 $x_1^2 + x_2^2 + x_3^2 = r^2 \leftarrow x_i x_i = r^2$

0,75 $9 = (3 \times 3) \leftarrow \delta_{ii} \delta_{jj}$

0,75 $3 = \sum_{i=1}^3 \delta_{ii} \leftarrow \delta_{ij} \delta_{jk} \delta_{ki}$

0,75 $\delta_{ij} \leftarrow \delta_{im} \delta_{mn} \delta_{nj}$

تمرين 2 (3pts)

$$\epsilon_{ijm} \epsilon_{kin} = \delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}$$

$$= \begin{vmatrix} \delta_{i1} & \delta_{i2} & \delta_{i3} \\ \delta_{j1} & \delta_{j2} & \delta_{j3} \\ \delta_{m1} & \delta_{m2} & \delta_{m3} \end{vmatrix} \begin{vmatrix} \delta_{1k} & \delta_{1l} & \delta_{1n} \\ \delta_{2k} & \delta_{2l} & \delta_{2n} \\ \delta_{3k} & \delta_{3l} & \delta_{3n} \end{vmatrix}$$

$$= \begin{vmatrix} \delta_{ik} & \delta_{il} & \delta_{im} \\ \delta_{jk} & \delta_{jl} & \delta_{jm} \\ \delta_{mk} & \delta_{ml} & \delta_{mn} \end{vmatrix}$$

$$= \delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}$$

تمرين 3 (10pts)

$$a \cdot (b \times c) = c \cdot (a \times b) = b \cdot (c \times a)$$

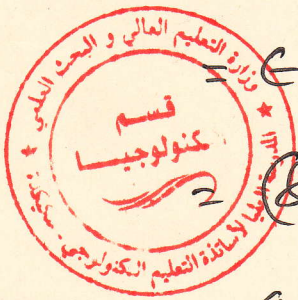
$$a \cdot (b \times c) = (a_i \vec{e}_i) \cdot (\epsilon_{jkm} b_j c_k \vec{e}_m) = \epsilon_{jkm} a_i b_j c_k (\vec{e}_i \cdot \vec{e}_m) = \epsilon_{jki} a_i b_j c_k$$

$$a \cdot (b \times c) = \epsilon_{ijk} a_i b_j c_k = \epsilon_{kij} c_k a_i b_j = c \cdot (a \times b)$$

$$= \epsilon_{jki} b_j c_k a_i = b \cdot (c \times a)$$

$$- a \times (b \times c) = (a \times c) \cdot b - (a \times b) \cdot c$$

$$a \times (b \times c) = a_i \vec{e}_i \times (\epsilon_{mjk} b_j c_k \vec{e}_m) = \epsilon_{mjk} a_i b_j c_k (\epsilon_{nin} \vec{e}_n)$$



$$= \epsilon_{jkn} \epsilon_{nin} a_i b_j c_k \vec{e}_n$$

$$= (\delta_{ji} \delta_{kn} - \delta_{ji} \delta_{kn}) a_i b_j c_k \vec{e}_n$$

$$= a_k b_n c_k \vec{e}_n - a_i b_i c_n \vec{e}_n$$

$$a \times (b \times c) = (a \times c) b - (a \times b) c$$

(4 P5) 4 نمر

$$\det A = \epsilon_{ijk} A_{i1} A_{j2} A_{k3}$$

$$= \epsilon_{1jk} A_{11} A_{j2} A_{k3} + \epsilon_{2jk} A_{21} A_{j2} A_{k3} + \epsilon_{3jk} A_{31} A_{j2} A_{k3}$$

$$= \epsilon_{123} A_{11} A_{22} A_{33} + \epsilon_{132} A_{11} A_{32} A_{23} + \epsilon_{231} A_{21} A_{32} A_{31}$$

$$+ \epsilon_{203} A_{21} A_{12} A_{33} + \epsilon_{312} A_{31} A_{12} A_{23} + \epsilon_{321} A_{31} A_{22} A_{13}$$

$$= A_{11} (A_{22} A_{33} - A_{32} A_{23}) + A_{21} (A_{32} A_{13} - A_{12} A_{33})$$

$$+ A_{31} (A_{12} A_{23} - A_{22} A_{13})$$

$$= \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}$$